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# ASPNORM:
# A Normal Diameter Distribution Growth and Yield Model for Aspen in the Central Rocky Mountains

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**Abstract**

Development of a normal diameter distribution growth and yield model for pure, even-aged, unthinned clones of aspen in the central Rocky Mountains is described, including testing for normality of diameter distributions, development of regression estimators for stand parameters, model validation, and model application.

[1] *Headquarters is in Fort Collins, in cooperation with Colorado State University.*

245

# ASPNORM: A Normal Diameter Distribution Growth and Yield Model for Aspen in the Central Rocky Mountains

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## Management Implications

ASPNORM is a computer program which calculates growth and yield for pure, even-aged, unthinned clones of aspen (*Populus tremuloides* Michx.) in the central Rocky Mountains. The model assumes that numbers of stems at each projection period are distributed across diameter classes according to the normal[2] probability distribution. This provides the manager with information on stand density and volume by size class. Yield tables are printed for initial stand conditions and for fixed 10-year cycles.

The model has been developed for the most practical and economical silviculture option for aspen. Because of disease and regeneration considerations, as well as mathematical continuity, no provision is made for thinnings or partial overstory removals. A complete harvest cut is assumed at the final projection period.

ASPNORM provides stand average values for basal area per acre, stems per acre, average stand height, mean and variance of diameter breast height, and total cubic foot, merchantable cubic foot, and board foot Scribner rule volumes. In addition, stems per acre, average heights, and the above volumes are listed based upon a normal probability distribution of stems per acre across 1-inch diameter classes. This provides a breakdown of potential stand products by merchantability class. An example of program output is shown in Appendix 1.

Initial stand values for the model may be provided either by a tree list or by stand average values. If a tree list of diameters is used, corresponding heights may be provided, or may be calculated by the program. Age, site index (Edminster et al. 1985), and the inverse of plot area are also required. An example of program initialization for this method is shown in Appendix 1. The diameter distribution is tested for normality with a very low probability of rejection. If diameters are not distributed normally, projections do not proceed. If initial stand diameters are known to be normally distributed, mean stand diameter, variance of diameter, stems per acre, age, and site index may be directly entered in the program. The program is written to interact with the user, asking for the appropriate information.

Diameter distribution models have the advantage of providing information on the numbers of stems across diameter classes, while estimating only a few stand average variables at each projection period. They combine advantages of whole stand distance-independent models which are relatively simple and require little computation time, with information on the distribution of diameters which is an advantage of individual tree type models (Munro 1974).

The normal diameter distribution provides a simpler approach to stand modeling than other distributions. The mean and variance are useful and easily understood distribution parameters. A normal distribution is always symmetric about the mean diameter, with the associated variance governing the relative rate of change in numbers of stems between adjacent diameter classes. The normal distribution is easily tested for adequacy of fit to a set of tree diameters.

The principal disadvantage of the normal diameter distribution approach is the lack of flexibility in the shape. When compared to the Weibull or Johnson's $S_B$ distributions, the lack of flexibility of distribution shape may be compensated for by the increased accuracy of mean and variance parameter prediction. The prescreening of stand diameter distributions for normality provides greater predictive accuracy over models which assume an underlying distribution for all stands. Normality and the resultant input data requirements limit ASPNORM's range of application, however.

ASPNORM is written in FORTRAN 77 computer language. It is available on 5.25-inch flexible disk for IBM compatible microcomputers configured with a minimum of 256 kilobytes of memory, one or more disk drives, and using the DOS 2.0 operating system. Output is printed in 132-column format. ASPNORM is also available on nine-track, half-inch magnetic tape for transfer to mainframe computers. Copies may be obtained by sending the author a disk or tape.

## Model Application

In the central Rocky Mountains, aspen grows as a clone of genetically identical trees (ramets) which result from adventitious sprouting of the widespread, shallow, parent root system. These sprouts become independent stems as they mature. ASPNORM predicts the growth of a clone of independent stems. Projections based upon measurements from juvenile clones in which stems are not yet independent may lead to erroneous conclusions about stand potential.

Aspen does not stagnate because of overpopulation as does lodgepole pine (*Pinus contorta* Dougl.). Rapid periodic natural thinning characterizes aspen stand development (Jones and Schier 1985). Shade intolerance, apical dominance, and clonal growth may all play a role in aspen's ability to self-thin (Schier et al. 1985). Even when logging damage is kept to a minimum, thinning or partial cutting operations leave residual stems more susceptible to fatal diseases (Jones and Shepperd 1985, Walters et al. 1982). Unless the total number of stems removed in a thinning was carefully apportioned across the diameter classes, the underlying assumption of a

[2]*The term normal refers to the statistical probability distribution commonly described as "bell-shaped", and should not be confused with normal stocking.*

normal distribution of stems across diameter classes would be violated. For these reasons, no options for intermediate thinnings or partial removal cuts are provided in ASPNORM.

The mortality functions in ASPNORM were developed to reflect average conditions over many healthy stands. They do not reflect losses resulting from short-term fluctuations in mortality or long-term decay. Decay loss is a serious problem in many aspen stands over 100 years (Hinds 1985, Shepperd and Engelby 1983). A pathological rotation of 90 to 120 years was recommended by Hinds and Wengert (1977) and should be considered as an upper bound on rotation length in ASPNORM.

### Stand Measurement

Each measured plot should be located so that only one clone is included. Point samples should not be used. The arithmetic mean diameter is the variable of interest for ASPNORM, not quadratic mean diameter. As a general rule, fixed area plots of 1/5 acre for mature (sawlog) stands, 1/20 acre for intermediate (pole) stands, and 1/50 acre for juvenile (sapling) stands should be sufficient to provide unbiased estimates of the mean and variance of diameter.

As with all growth models, the stand area being modeled must be homogeneous with respect to stand variables and clonal boundaries. If not, areas should be stratified to maintain this homogeneity as well as to eliminate any unproductive areas (Avery and Burkhart 1983). All volumes are based upon the merchantability standards used in the aspen volume tables developed by Edminster et al. (1982). No allowance for decreased yield resulting from defect, disease, or harvesting losses has been made. Volume estimates assume that all material meeting minimum merchantability standards will be utilized.

ASPNORM should only be applied to biologically reasonable combinations of stand variables which lie within the following ranges of calibration data:

Mean d.b.h. from 0.8 to 13.8 inches,
Variance of d.b.h. from 0.13 to 21.04 inches squared,
Stems per acre from 143 to 5738,
Stand age from 10 to 121 years, and
Site index from 29 to 99 feet.

Frequencies of driving variables from ASPNORM calibration data are shown in table 1. ASPNORM should not be applied to areas outside Colorado, southern Wyoming, and eastern Utah without additional validation or recalibration using the procedures outlined below.

### Data Collection and Preparation

One hundred temporary plots of 100 to 150 aspen trees were selected from eight national forests on the western slope of the central Rocky Mountains. The plots of even-aged aspen were placed to encompass only a single clone. Areas with defect and disease which could affect growth or with more than 10% of the overstory stems in species other than aspen were not measured. A wide range of values were obtained for stand diameter, stand height, number of stems per acre, basal area, site index, and stand age to preclude unintentional correlation between these variables. Field procedures followed those recommended by Vuokila (1964) for temporary sample plots.

Table 1.—Frequencies of driving variables from ASPNORM calibration data.

| Stand Mean Diameter | | Variance of Diameter | | Stems per Acre | | Stand Mean Age | | Site Index | |
|---|---|---|---|---|---|---|---|---|---|
| Diameter Class | Freq. | Variance | Freq. | Stems | Freq. | Age | Freq. | Site | Freq. |
| 0.1- 1.0 | 1 | 0.01- 0.50 | 9 | 1- 150 | 1 | 1- 10 | 4 | 30 | 2 |
| 1.1- 2.0 | 8 | 0.51- 1.00 | 12 | 151- 300 | 7 | 11- 20 | 8 | 40 | 8 |
| 2.1- 3.0 | 5 | 1.01- 1.50 | 12 | 301- 450 | 10 | 21- 30 | 2 | 50 | 8 |
| 3.1- 4.0 | 14 | 1.51- 2.00 | 9 | 451- 600 | 7 | 31- 40 | 10 | 60 | 18 |
| 4.1- 5.0 | 11 | 2.01- 2.50 | 11 | 601- 750 | 12 | 41- 50 | 2 | 70 | 13 |
| 5.1- 6.0 | 8 | 2.51- 3.00 | 8 | 751- 900 | 4 | 51- 60 | 12 | 80 | 18 |
| 6.1- 7.0 | 10 | 3.01- 3.50 | 4 | 901-1050 | 14 | 61- 70 | 13 | 90 | 14 |
| 7.1- 8.0 | 8 | 3.51- 4.00 | 1 | 1051-1200 | 3 | 71- 80 | 14 | 100 | 2 |
| 8.1- 9.0 | 7 | 4.01- 4.50 | 3 | 1201-1350 | 4 | 81- 90 | 13 | | |
| 9.1-10.0 | 2 | 4.51- 5.00 | 2 | 1351-1500 | 1 | 91-100 | 3 | | |
| 10.1-11.0 | 4 | 5.01- 5.50 | 1 | 1501-1650 | 2 | 101-110 | 1 | | |
| 11.1-12.0 | 2 | 5.51- 6.00 | 4 | 1651-1800 | 1 | 121-130 | 1 | | |
| 12.1-13.0 | 2 | 7.01- 7.50 | 2 | 1801-1950 | 1 | | | | |
| 13.1-14.0 | 1 | 8.01- 8.50 | 1 | 1951-2100 | 3 | | | | |
| | | 8.51- 9.00 | 1 | 2101-2250 | 4 | | | | |
| | | 10.51-11.00 | 1 | 2851-3750 | 2 | | | | |
| | | 11.01-11.50 | 1 | 3751-3900 | 1 | | | | |
| | | 21.01-21.50 | 1 | 4051-4200 | 2 | | | | |
| | | | | 4201-4350 | 1 | | | | |
| | | | | 4351-4500 | 1 | | | | |
| | | | | 5551-5700 | 1 | | | | |
| | | | | 5701-5850 | 1 | | | | |

All trees were measured for diameter at breast height; bark thickness and 10-year radial growth were measured using techniques similar to those reported by Jones (1966). One tree in each 1-inch diameter class was cored through the pith for age. Total height was measured to the nearest foot for five trees in each diameter class. Site index was calculated from the Edminster et al. (1985) site index function for three to five of the most vigorous stems in each plot.

Diameter, height, and radial wood growth information was used to estimate conditions 10 years ago for each tree individually as outlined by Myers (1971). Diameters and heights were calculated for all trees in each plot at the time of measurement and estimated for 10 years before measurement. Past diameters were estimated using a diameter reconstruction equation (Mowrer and Edminster 1985). Past heights were estimated using a diameter-height regression calibrated to each plot.

Time of death was determined for each dead tree based upon stem condition. Diameter was measured for those dying within the previous 10 years. Change in diameter was considered to be negligible for trees dying within the 10-year mortality period. Past heights were estimated for these diameters using the plot diameter-height regression. These trees were included in the 10-year past stand estimate.

## Applying the Normal Distribution

The log likelihood and several other goodness of fit statistics were calculated using the procedures in Schreuder et al. (1978) to fit six probability distribution functions to a data set. Those tested were the normal, lognormal, gamma, Weibull, Johnson's $S_B$, and beta distributions. Using the log likelihood statistic, Johnson's $S_B$ best fit the data from the 100 even-aged aspen plots. However, it was not possible to find adequate parameter prediction equations for Johnson's $S_B$ using stand parameters as independent variables. Reexamination of the log likelihood values showed that the normal distribution performed almost as well as the $S_B$ distribution.

Therefore, the normal distribution was tested for an adequate fit. Fisher's kappa statistic test (Bliss 1967) was used as the basis for a FORTRAN 77 program which tested all diameter values in each plot for normal skewness and kurtosis. A low probability level ($P = 0.01$) was used for the test. This low level allowed 83 plots to be classified as having normally distributed diameters. Use of a low probability level increased the chance of failing to reject a data set derived from a non-normal population. It was selected in order to create a model with a wider range of application. If only one of skewness or kurtosis was significantly non-normal, then the chi-square test recommended by Bliss (1967) was used. This program for testing normality became the basis for subroutine TESTIT in the growth model.

Diameter at breast height versus age was graphed for all of the 17 significantly non-normal plots and for a representative sample of the 83 plots with normally distributed diameters. In every non-normal case, there was a group of several trees which were separated from the others by several inches of diameter and at least 10 years in age. This indicated a departure from a true even-aged condition, as evidenced by the more gradual transitions across diameter and age for the normally distributed diameters.

## Development of Regression Estimators

Regressions were developed using BMDP statistical software regression packages (Dixon 1981) for mean diameter in 10 years ($D_{10}$), variance of diameter in 10 years ($V_{10}$), and stems per acre in 10 years ($STM_{10}$). Estimators for current tree height ($HT_0$) as a function of current tree diameter ($D_0$) and stand conditions also were developed for low, medium, and high site index classes. Transformations involving square roots ($X^{1/2}$), squares ($X^2$), natural logarithms ($LnX$), and the squares of natural logarithms ($(LnX)^2$) were explored, as well as their reciprocals.

Care was taken to insure that the magnitude and sign of the coefficient of each independent variable included in regression equations made biological sense in their effect upon the quantity predicted. Because cross-product terms, particularly when involving transformations, were difficult to assess biologically, they were not used. The number of independent variables was kept to a minimum. Each term included in the equations provided a sizable reduction in $R^2$ and standard error. Unless they were highly significant, two forms of a variable were not allowed.

Separate equations for different age or site classes were used for prediction of mortality, variance, and height. Fitting separate functional relations to these groups improved model performance over a single equation. Four criteria were used to judge these types of equations: (1) the fit of the separate equations, (2) smooth transition between equations (mortality only), (3) individual function sensitivity analysis to insure biologically reasonable predicted values across the range of calibration data, and (4) whole model sensitivity analysis of mortality and height estimates and of periodic and mean annual volume increment relationships.

### Prediction of Mean Diameter in 10 Years

Bailey (1980) showed that a normal diameter distribution implied future diameter was a linear function of past diameter. The mean diameter prediction equation was thus developed using future mean diameter ($D_{10}$) as the dependent variable and current diameter ($D_0$) as an independent variable. Other independent variables from past stand conditions were investigated, including basal area (B.A.), stems per acre (STM), age (AGE), and site index (S.I.), as well as their transforms mentioned above. The final regression equation in the model was,

$$D_{10} = 0.14 + 1.03(D_0) - 0.0042(AGE) + 4.1/Ln(STM)$$
$$(R^2 = 0.9973, S_{y.x} = 0.1584).$$

### Mortality Prediction

Because different biological controls and influences affect mortality as stand age increases, mortality was estimated for three breast height age ranges: juvenile (10 to 30 years), intermediate (31 to 70 years), and mature (greater than 70 years). Some overlap of age ranges was allowed in the three data sets to insure a smooth transition between functions. Future stems per acre ($STM_{10}$) were predicted as a function of current stems per acre ($STM_0$) and variables reflecting past stand conditions. Possible independent variables included mean diameter at breast height ($D_0$), variance of diameter at breast height ($V_0$), basal area (B.A.), age (AGE), and site index (S.I.). For ages up to 30 years at breast height, the regression equation was,

$$STM_{10} = 940.7 + 0.999(STM_0) - 20580.0/(\mathrm{Ln}(S.I.))^2$$
$$(R^2 = 0.9874,\ S_{y.x} = 172.86,\ N = 20 \text{ stands}).$$

As the value of site index changes from 90 to 40 feet, 10-year mortality is increased by 496 stems, reflecting a decrease in juvenile survival for poorer site aspen stands. For estimation from 30 to 70 years, the equation was,

$$STM_{10} = 13.0 + 0.936(STM_0) - 72.25/(V_0^2) - 0.0000341(STM_0^2)$$
$$(R^2 = 0.9838,\ S_{y.x} = 85.4595,\ N = 44 \text{ stands}).$$

Here, the value of current variance ($V_0$) reduces the number of stems per acre by 380 for smallest variance encountered in the younger stands, but affects mortality by less than one stem for the largest variance value in older stands. For estimation with stand age greater than 70 years, the equation was,

$$STM_{10} = 11.7 + 0.889(STM_0) - 68.99/(V_0^2)$$
$$(R^2 = 0.9863,\ S_{y.x} = 38.0510,\ N = 39 \text{ stands}).$$

Variance can again be seen to have an inverse effect upon mortality. Figure 1 shows the number of stems per acre by diameter class for an average stand at 20-year intervals from ages 10 to 90 years.

### Prediction of Variance of D.B.H. in 10 Years

Future variance of diameter at breast height ($V_{10}$) was estimated using current variance of diameter at breast height ($V_0$) as the primary independent variable. Other independent variables similar to those used for diameter prediction were considered. Simple linear regressions for two age ranges coinciding with the first and the last two mortality equations best reflected the changes in variance. The variance prediction equation for stands in the same age range as the juvenile mortality equation was,

$$V_{10} = 0.151 + 1.150(V_0)$$
$$(R^2 = 0.9655,\ S_{y.x} = 0.1471).$$

The second variance prediction equation developed from data for intermediate and mature stands was,

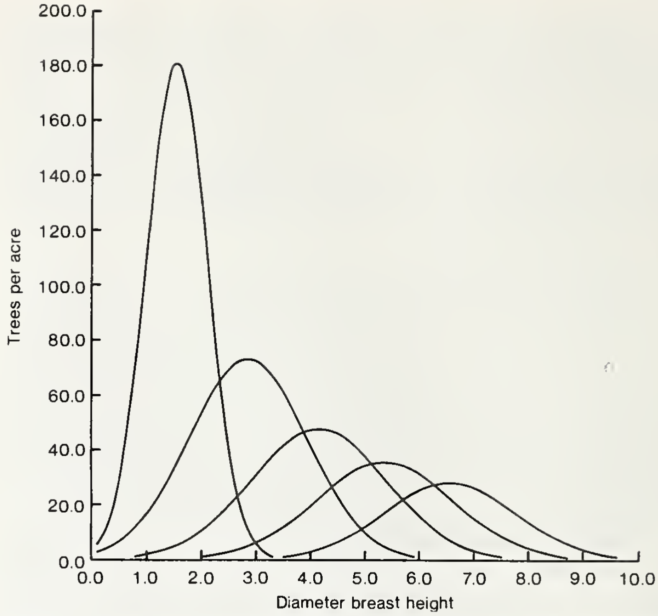


**Figure 1.—Numbers of stems per acre at 20-year intervals from ages 10 to 90 (left to right).**

$$V_{10} = 0.064 + 1.014(V_0)$$
$$(R^2 = 0.9759,\ S_{y.x} = 0.5167).$$

These equations reflect a faster increase in the range of distribution of diameters in younger aspen stands and the slowing of this trend in established stands.

### Height Prediction

Individual tree observations were separated into three groups by site class. Current height (HT) was estimated for low sites (less than site 50), medium sites (sites 50 through 79), and high sites (80 and higher). The final regression developed to predict current height for sites less than 50 was,

$$\mathrm{Ln}(HT - 4.5) = 0.77 + 1.49(\mathrm{Ln}(D)) - 0.25(\mathrm{Ln}(D))^2 + 0.00064(B.A.) + 0.0056(AGE) + 0.00016(S.I.)^2 + 0.000000015(STM)^2$$
$$(R^2 = 0.9064,\ S_{y.x} = 0.1319,\ N = 1684 \text{ Observations}).$$

For sites from 50 to 79 feet the regression was,

$$\mathrm{Ln}(HT - 4.5) = 1.05 + 1.49(\mathrm{Ln}(D)) - 0.21(\mathrm{Ln}(D))^2 + 0.00025(B.A.) + 0.0044(AGE) + 0.00007(S.I.)^2 + 0.000000013(STM)^2$$
$$(R^2 = 0.9349,\ S_{y.x} = 0.1581,\ N = 4966 \text{ Observations}).$$

For sites greater than or equal to 80,

$$\mathrm{Ln}(HT - 4.5) = 1.13 + 1.48(\mathrm{Ln}(D)) - 0.21(\mathrm{Ln}(D))^2 + 0.00054(B.A.) + 0.0029(AGE) + 0.00007(S.I.)^2 + 0.000000004(STM)^2$$
$$(R^2 = 0.9681,\ S_{y.x} = 0.1185,\ N = 1693 \text{ Observations}).$$

Figure 2 shows estimated tree height for 10-foot site classes from 40 through 90.

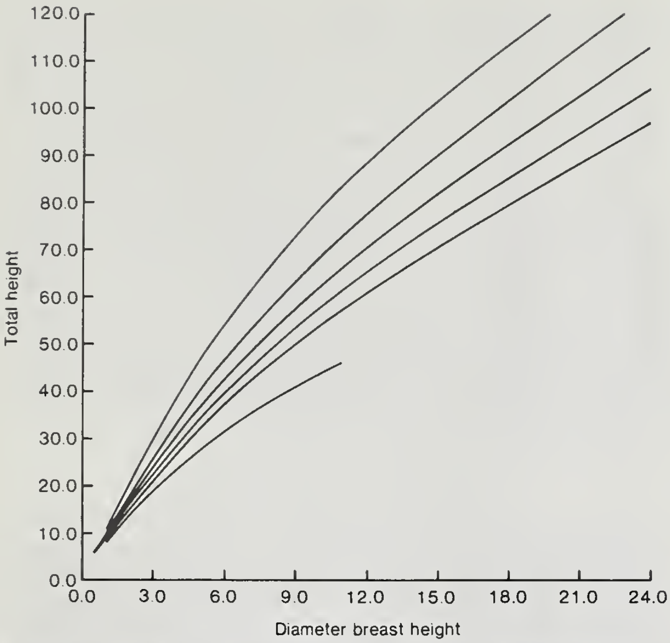


**Figure 2.—Tree height estimation for 10-foot site classes from 40 (bottom curve) to 90 (top curve).**

## Volume Prediction

Volume prediction was based upon the Edminster et al. (1982) aspen volume equations for gross volume, in cubic feet inside bark, of the entire stem including stump and top; for gross merchantable volume, in cubic feet inside bark, of the merchantable stem excluding stump and top; and for gross volume, in board feet, inside bark, Scribner rule, of the merchantable stem excluding stump and top.

## ASPNORM Model Operation

ASPNORM was written in FORTRAN 77 using a modular format utilizing one main program whose principal function is to call the 11 subroutines in proper sequence. Subroutines provide for data input, normality testing, calculation of theoretical normal stand conditions, height and volume calculations, 10-year growth projections, and output of variable density stand tables for initial stand data and theoretical normal stand conditions for each projection period. During program documentation, every effort was made to define each variable in the subroutine in which it was first used and to insert adequate comments to provide the interested user with enough information to follow the flow of logic within the program. Figure 3 shows a block diagram of ASPNORM program operation.

Overall model operation consists of two main program sequences. The model first compares actual and theoretical conditions for the initial time period. If the diameters are normally distributed, the model then proceeds into the main growth projection loop. Both the initial and growth projection portions use many of the same subroutines. The driving variables for the program are mean diameter, variance of diameter, blow up factor

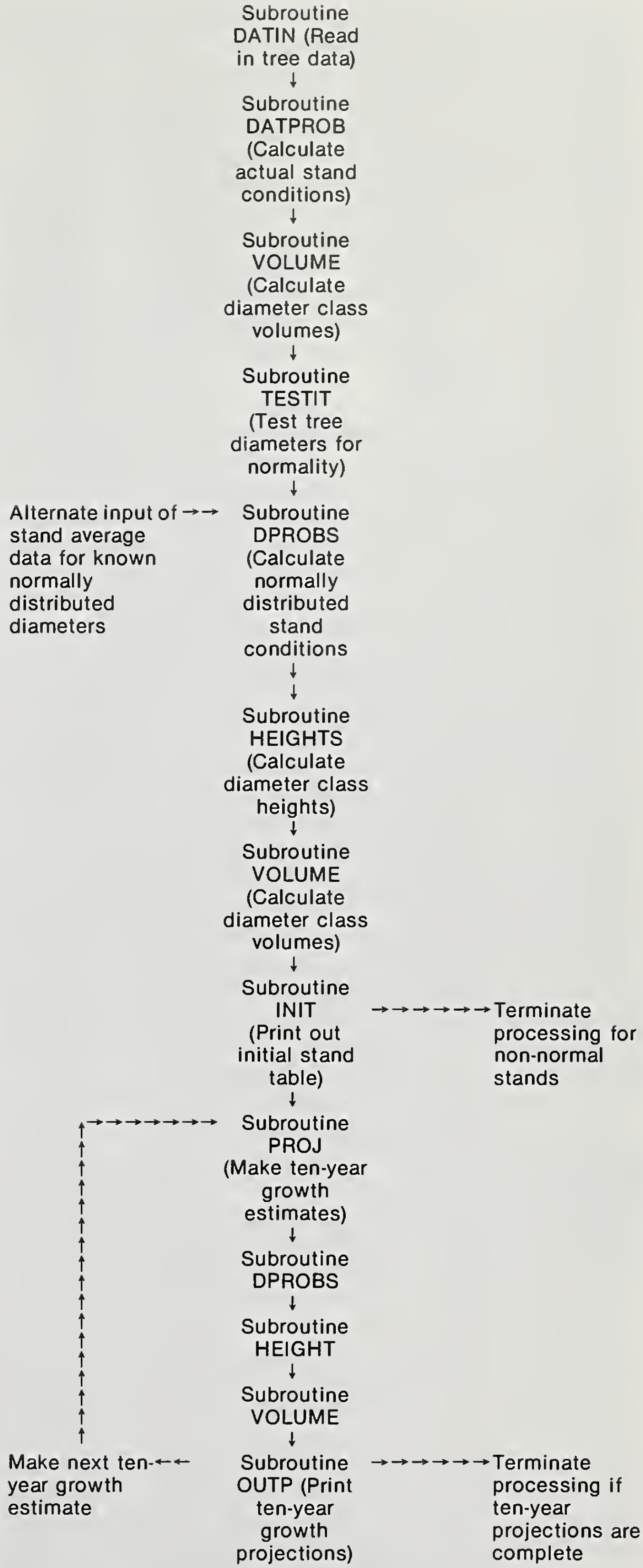


**Figure 3.—Block diagram of ASPNORM program operation.**

(the number of trees per acre represented by one plot tree, or the inverse of stems per acre), average plot age, and site index. This information may be provided in two

ways: either by actual plot data, including individual tree diameters which will be tested for normality, or by introducing the five variables directly for stands which are known to have a normal diameter distribution.

In the first method, the first step is to read the stand data consisting of diameter and height pairs for each tree, plot site index (Edminster et al. 1985), average plot age, and plot size. In addition, the number of 10-year projection periods for each stand must be initialized. This is accomplished by subroutine DATIN. Alternatively, mean and variance of diameter, number of stems per acre, site index, and stand average age may be directly initialized for stands with normally distributed diameters.

Subroutine DATPROB is next called to calculate the values for actual stand conditions from the data read by DATIN. Proportions and numbers of stems per acre are calculated by 1-inch diameter classes, diameter class zero running from 0.1 to 1.0 inches, diameter class one running from 1.1 to 2.0 inches, up through diameter class 39 running from 39.1 to 40.0 inches. Subroutine VOLUME is called to calculate tree volumes for the actually measured diameters and heights in each diameter class using equations from the Edminster et al. (1982) volume tables. Actual stems per acre, basal area, average height, and volumes are calculated for the stand as a whole. Average tree heights, numbers of stems per acre, and tree volumes are calculated by diameter class. Minimum and maximum tree diameter is noted and control is returned to the main program.

Subroutine TESTIT uses Fisher's kappa test for skewness and kurtosis to test the plot diameter data for distributional normality. In the course of calculating the test statistics, the mean diameter at breast height and the variance of diameter at breast height are computed. After calculating the test statistics as previously described, a flag is set which continues processing of the stand for normally distributed diameters, or terminates processing of the stand for non-normal diameters.

In subroutine DPROBS, the values for mean diameter at breast height and the associated variance are used to calculate the stand values based upon a normal probability distribution. Standard normal deviates are calculated for each diameter class upper and lower limit, and the cumulative density function is evaluated between these limits using a Romberg integral approximation method. Associated stems per acre by diameter class are determined and summed to estimate the total stems per acre for the whole stand. Diameter classes are rounded up to the next whole tree if they contain a fraction of 0.5 or greater. Subroutines HEIGHT and VOLUME are called for calculations by diameter class. Total stand basal area, stems per acre, average height, and stand volumes for the theoretical normal stand are summed over diameter classes. Minimum and maximum diameter classes for the normal distribution are noted. If the five driving variables are to be initialized for known normal stands, they are introduced directly to this subroutine. Previous subroutine calls are then modified.

The final step for the initial program sequence is to print out a table of initial actual and theoretical normal values for numbers of trees, diameters with associated heights and volumes, and stand total values by subroutine INIT. If the stand diameters are normally distributed, the program then enters the main loop to make the specified number of 10-year projections.

The first step in the projection cycle is for subroutine PROJ to make 10-year projections for mean diameter, variance of diameter, and number of stems per acre, and to increment stand age by 10 years. DPROBS is called again to calculate the same variables for the updated driving variables. Subroutine HEIGHT and VOLUME are called. Subroutine OUTP is called to print the table of current stand conditions. If the 10-year projections are complete, processing is terminated. Otherwise, a new 10-year growth projection is made by PROJ.

## ASPNORM Model Validation

Two approaches were used to analyze the accuracy of model predictions. First, sensitivity analysis was used to assess the effect of changes in the initial values of the driving variables upon predicted output variables for mean diameter, variance of diameter, stems per acre, average stand height, basal area per acre, and total, merchantable, and board foot Scribner volumes. Second, the accuracy of model projections was evaluated by recalibrating the model on a subset of the data and comparing the values for one 10-year projection with the remaining actual 10-year plot data.

### Sensitivity Analysis

To generate a realistic series of model input vectors for sensitivity analysis, the lower age quartile of the plot averages from the calibration data was used to calculate a mean vector and variance-covariance matrix for the driving variables in the model. This, in turn, determined a multivariate normal distribution used to simulate vectors of the driving variables representing the naturally occurring interrelationships between mean diameter, variance of diameter, stems per acre, average plot age, and site index.

A FORTRAN 77 program was written which used this multivariate normal relationship to create a series of five sets of 100 random vectors of driving variables. In each set, for one of the driving variables the coefficient of variation (the standard deviation divided by the mean) was 0.10. The coefficient of variation (c.v.) provides a unitless measure useful in comparison of the relative variability in each element. The other driving variables only varied proportionately to the one with the established c.v. as dictated by the variance-covariance relationship.

These five sets of 100 vectors were used as input to the model to initiate five 10-year projections. Coefficients of variation calculated from the initial vectors and the resulting projections are shown in the six columns of table 2. Only the table for fixed c.v. of mean diameter at breast height is shown here since the trends were similar for the remaining four tables. The 10-year

Table 2.—ASPNORM sensitivity analysis: coefficient of variation for model variables over five 10-year projections.

| | | | 10-Year Projection | | | |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 |
| Mean d.b.h. | 0.10 | 0.09 | 0.08 | 0.07 | 0.07 | 0.06 |
| Variance of d.b.h. | 0.16 | 0.17 | 0.18 | 0.20 | 0.23 | 0.27 |
| Stems per acre | 0.19 | 0.17 | 0.15 | 0.14 | 0.13 | 0.14 |
| Mean age | 0.09 | 0.07 | 0.06 | 0.05 | 0.04 | 0.04 |
| Site index | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| Basal area per acre | 0.03 | 0.02 | 0.02 | 0.02 | 0.03 | 0.04 |
| Mean height | 0.09 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 |
| T.c.f. volume | 0.08 | 0.08 | 0.07 | 0.07 | 0.07 | 0.07 |
| M.c.f. volume | 0.46 | 0.33 | 0.24 | 0.18 | 0.14 | 0.10 |
| B.f.s. volume | 1.09 | 0.90 | 0.76 | 0.64 | 0.52 | 0.40 |

projection column headed by a zero shows the initial input values for the c.v. of each of five driving variables. Coefficients of variation for stand variables which were calculated from them are shown in the last five rows. Each 10-year projection column shows the coefficient of variation for each variable at successive projection periods.

In general, a decreasing trend in c.v. across columns indicates increasing stability in that variable over time, as the standard deviation increases more slowly than the mean. Variables which show an increasing trend across columns evidence instability over time. A constant c.v. over projection periods indicates insensitivity of that variable to changes in input variables.

Initial values of both variance of diameter at breast height and stems per acre are highly correlated with changes in mean diameter at breast height in the multivariate normal setting in which the initial input variables were generated. This is an artifact of the calibration data which was used to generate the variance-covariance matrix. Comparison of the initial values for the c.v. of volumes across tables for fixed values of the other input variables showed a greater sensitivity to changes in variance of diameter at breast height, mean diameter at breast height, and stems per acre in that order.

Examination of the trend in mean diameter across projections shows a stable and decreasing trend. Variance, however, shows just the opposite trend. An increase in the c.v. of variance is more apparent for higher initial values, also. Stems per acre shows an initial decrease followed by an increase in the fifth projection period. Mean stand age exhibits a stable decreasing trend across all periods. Site index remains constant for all periods.

## Validation by Data Splitting

After the final form of the growth regressions had been determined, model performance was assessed by splitting the calibration data, recalibrating the model on the larger portion of the data, and testing the predictive ability of the model on the remainder. For validation of ASPNORM, a subset of 21, or a little more than 25% of the plots, were deleted from the total of 83. After deletion of the validation subset, the prediction regressions for mean diameter at breast height, variance of diameter at breast height, stems per acre, and height were recalculated with the same functional form used for the entire data set.

As previously described, actual stand measurements were used to calculate individual tree and stand average conditions 10 years prior to measurement. For the verification subset, these prior values initialized the driving variables for the model. One 10-year projection for these 21 plots gave a set of predicted current values resulting from the forward projection of the calculated prior values to compare with the observed current values which were directly measured as stand conditions on the plots. Thus, a set of verification data consisting of actual and estimated values for mean diameter at breast height, variance of diameter at breast height, stems per acre, basal area per acre, average height, total cubic foot, merchantable cubic foot, and Scribner rule board foot volume was generated. Values for the regression coefficients for the 62 recalibration plots were consistent with those from the entire set of 83 plots.

As a method of comparison of observed versus predicted stand values, regressions were calculated with the model-estimated value for each stand parameter as the dependent variable and the actual stand value calculated from measured tree data as the independent variable. The general form of the regression is:

$$\text{Predicted value} = b_0 + b_1 \text{ Observed value},$$

where $b_0$ and $b_1$ are the regression coefficients determining the intercept and the slope of the linear equation.

A regression equation indicating a perfect model prediction would have a slope of 1 and an intercept of 0, thus creating a line through the origin with a slope of 45°. Portions of the regression equations which lie above this 45° line indicate a trend toward model overestimation, because values predicted by the model are greater than observed values. Conversely, portions of the regression equations below the 45° line indicate underestimation.

In order to provide an indication of whether the intercept and slope coefficients reasonably approximate the ideal values of 0 and 1, 90% simultaneous confidence intervals were calculated using the technique outlined in Draper and Smith (1981). If the calculated coefficients ($b_0$ and $b_1$) lie within the confidence region about 0 and 1, one may be 90% confident that they provide jointly reasonable estimates for the ideal values. This method accounts for correlation between the calculated coefficients which would not be considered by individual confidence intervals about each coefficient separately. The calculated coefficients and the 90% confidence interval results are shown in table 3.

Tests recommended by Reynolds (1984) rejected normality of the deviations of observed minus predicted values from the data splitting validation. This may be because of the small sample size of 21. Statistics used for comparing model accuracy recommended by Reynolds (1984) were, therefore, not calculated. Failure of tests for a mean of 0 for the deviations indicated bias was present in the model calibrated with the validation subset of 62 stands.

Arney (1984) recommended plotting the mean for groups of residuals (observed minus predicted values) across the range of observed values. Error bars showed the range of residuals for each group within plus and minus 2 standard deviations. Similar plots for the ASPNORM validation model are shown in figures 4 and 5. The residuals were sorted in ascending order by observed value and separated into three equal groups of seven. Means and standard deviations were calculated for each group, corresponding to low, middle, and high ranges for observed values. Differing scales on both axes should be considered in comparing trends in different variables. An upward trend for mortality prediction, and downward trends in volume prediction, show tendencies toward under- and overestimation, respectively, though error bars include 0 values in all cases.

## Conclusion

Sensitivity analysis indicates both the variance estimation and mortality portions of the model show predictive instability for long projections. The projection period at which this occurs depends upon the accuracy of the input. Users should consider increasing the accuracy of the variance of diameter at breast height and stems per acre estimates used to initialize the model.

The extensive error analysis of ASPNORM provides the user with an unusual amount of information on the predictive ability of the model. A trend toward overprediction of volume may be apparent in the validation model. This may be the result of the normal distribution which predicts the occurrence of large stems because of the untruncated upper tail of the distribution. These few large stems contribute a substantial proportion to total stand volume.

This bias may be less serious in the overall model which is calibrated with the entire set of 83 stands, instead of the subset used for the validation model. Independent validation data is required to determine this. In addition, the data splitting portion of the validation does not utilize independent data and only covers one 10-year projection. Both of these may tend to minimize the estimated model error. Additional variance analysis methods for ASPNORM are discussed by Mowrer (1984).

The model, as it stands, represents the best combination of estimative procedures for the data at hand. Attempts to model smaller and more numerous biological units result in a larger variance than that associated with the mean values of those units. Increased predictive error may thus be the price of obtaining additional diameter class information from stand average variables using the diameter distribution approach.

Table 3.—Verification regression coefficients and confidence interval results.

| Variable | Intercept ($b_0$) | Slope ($b_1$) | Are coefficients included in a 90% simultaneous confidence interval about the values of 0.0 and 1.0? |
|---|---|---|---|
| Diameter at breast height | −0.04671 | 1.00520 | yes |
| Variance of diameter at breast height | 0.13600 | 0.96560 | yes |
| Stems per acre | 38.86536 | 0.93891 | no |
| Basal area per acre | −12.55667 | 1.07479 | yes |
| Average stand height | 0.09298 | 1.02643 | yes |
| Total cubic foot volume per acre | −196.08447 | 1.10899 | no |
| Merchantable cubic foot volume per acre | −46.29028 | 1.08460 | no |
| Board foot volume (Scribner rule) per acre | −138.22363 | 1.08991 | no |

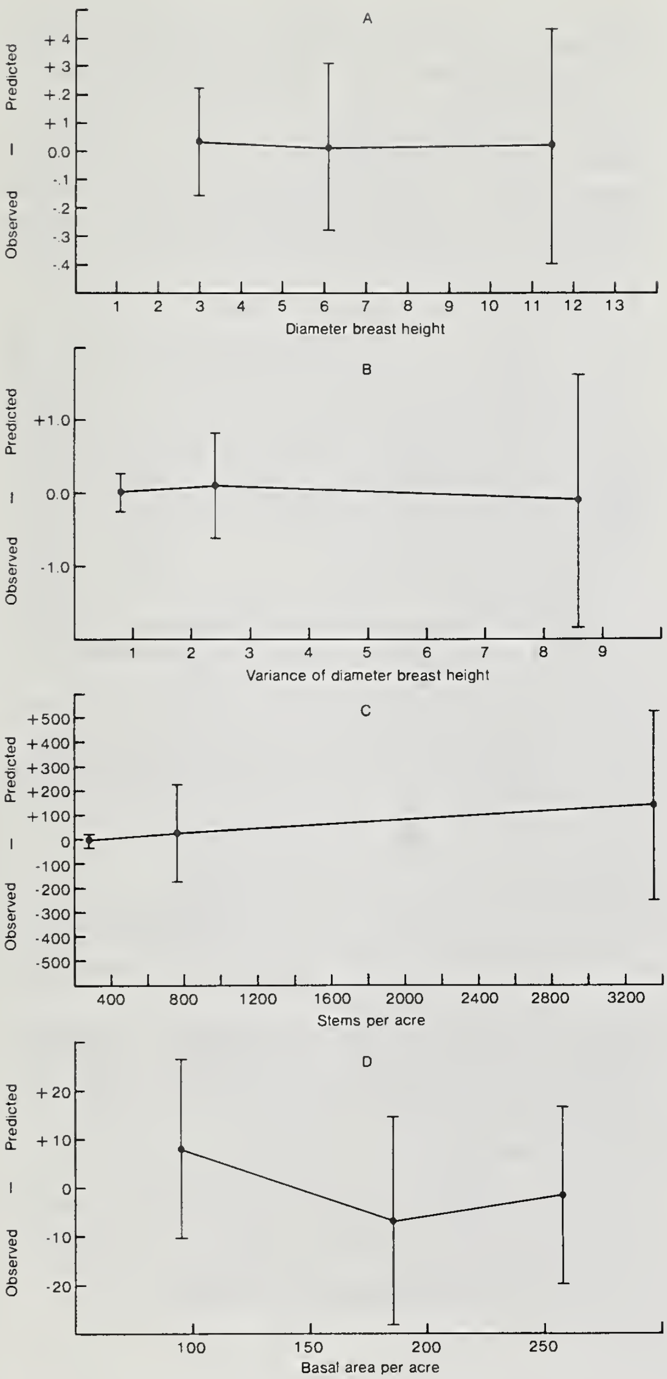


**Figure 4.—Residual ranges for mortality, diameter, basal area, and variance validation.**

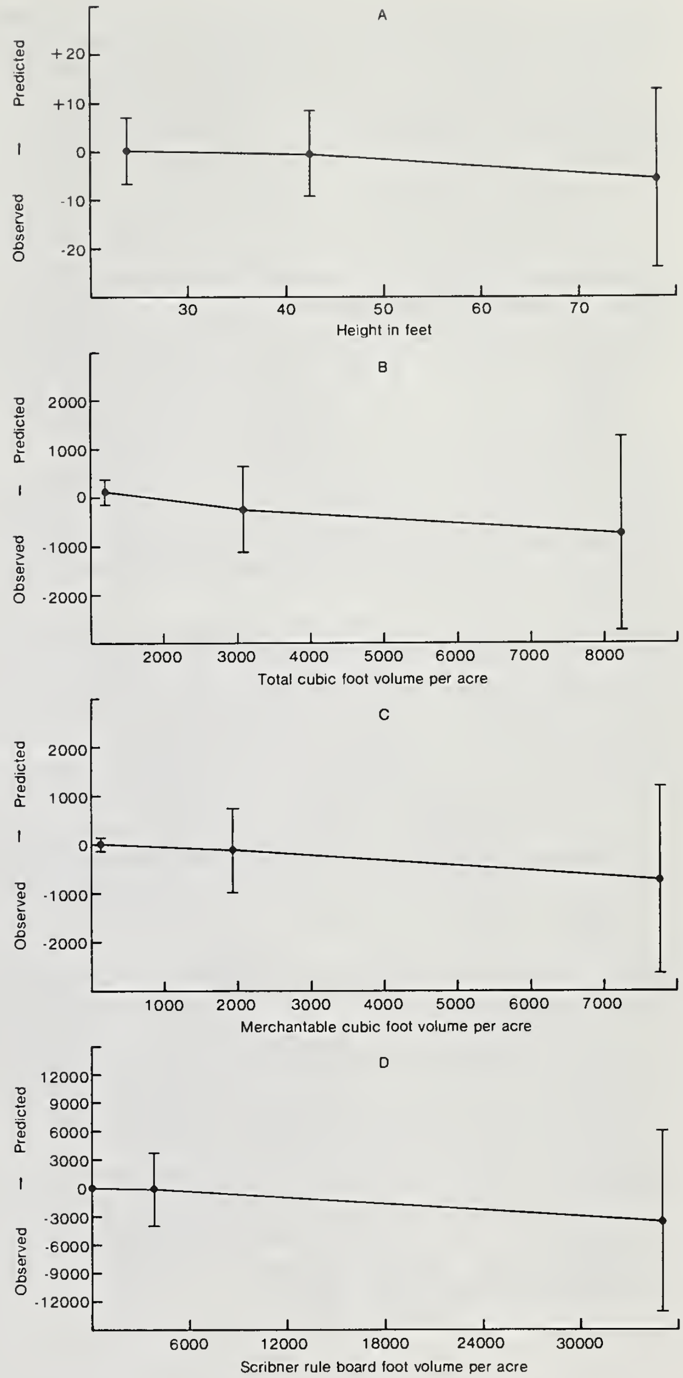


**Figure 5.—Residual ranges for height and volume validation.**

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## Appendix 1

### Program Initialization and Sample Output

ASPNORM may be initialized with either individual tree diameter data which will be tested for normality or with stand average data if the distribution of diameters is known to be normal. When individual tree information is provided, the following program sequence will occur.

WELCOME TO ASPNORM!

USE FREE FORMAT FOR ALL NUMBERS TO BE ENTERED MANUALLY. NUMBERS MAY BE SEPARATED BY BLANKS OR COMMAS. DECIMAL POINTS ARE NECESSARY FOR FLOATING POINT VARIABLES BUT CAN NOT BE USED FOR INTEGER VARIABLES.

ENTER PLOT TITLE (NOT TO EXCEED 80 CHARACTERS).

*TEST RUN WITH TEST.DAT INPUT DATA*

ENTER NUMBER OF 10-YEAR PROJECTIONS TO BE MADE (INTEGER).

*1*

ENTER AGE, SITE INDEX, AND BLOWUP FACTOR (FLOATING POINT).
*44.,82.,9.680*

DO YOU WISH TO PROJECT STANDS FROM A TREE LIST AND TEST FOR A NORMAL DIAMETER DISTRIBUTION, OR TO ENTER VARIABLES FOR STANDS WITH DIAMETERS WHICH ARE KNOWN TO BE NORMALLY DISTRIBUTED? ENTER T FOR TREE LIST, ELSE ENTER N.

*T*

TREE DATA MUST CONSIST OF ONE DIAMETER HEIGHT PAIR PER LINE. DIAMETERS SHOULD BE TO THE NEAREST TENTH INCH, HEIGHTS TO THE NEAREST FOOT.

INPUT FILE NAME?

*A:TEST.DAT*

ENTER INPUT FORMAT (ENCLOSED BY PARENTHESES).

*(F4.1,F4.0)*

119 ELEMENTS READ FROM FILE A: TEST.DAT

DATA ENTRY IS COMPLETE. STAND PROCESSING WILL TAKE A FEW MOMENTS.

CALCULATIONS FOR THE INITIAL PERIOD ARE COMPLETE.

CALCULATIONS FOR PROJECTION 1 ARE COMPLETE.

OUTPUT WRITTEN TO FILE "ASPNORM.OUT".

Stop – Program terminated.

STAND TITLE: TEST RUN WITH TEST.DAT INPUT DATA

| DIAMETER CLASS | ACTUAL PROPORTION | NORMAL PROBABILITY | ACTUAL STEMS/AC. | PREDICTED STEMS/AC. | ACTUAL HEIGHT | PREDICTED HEIGHT | ACTUAL T.C.F. | PREDICTED T.C.F. | ACTUAL M.C.F. | PREDICTED M.C.F. | ACTUAL B.F.S. | PREDICTED B.F.S. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .000000 | .007731 | 0 | 9 | 0. | 6. | .0 | .0 | .0 | .0 | 0. | 0. |
| 1 | .033613 | .036943 | 39 | 43 | 13. | 15. | 2.6 | 3.3 | .0 | .0 | 0. | 0. |
| 2 | .210084 | .112292 | 242 | 129 | 22. | 24. | 73.6 | 43.8 | .0 | .0 | 0. | 0. |
| 3 | .126050 | .217303 | 145 | 250 | 32. | 33. | 126.1 | 222.5 | .0 | .0 | 0. | 0. |
| 4 | .243697 | .267854 | 281 | 309 | 41. | 40. | 514.2 | 554.5 | .0 | .0 | 0. | 0. |
| 5 | .235294 | .210333 | 271 | 242 | 46. | 46. | 841.3 | 750.0 | 586.1 | 522.1 | 0. | 0. |
| 6 | .117647 | .105205 | 136 | 121 | 51. | 52. | 648.5 | 585.1 | 517.9 | 468.8 | 0. | 0. |
| 7 | .033613 | .033499 | 39 | 39 | 56. | 56. | 273.8 | 274.0 | 235.4 | 235.6 | 605. | 606. |
| 8 | .000000 | .006784 | 0 | 8 | 0. | 56. | .0 | 72.3 | .0 | 64.2 | 0. | 207. |
| 9 | .000000 | .000874 | 0 | 1 | 0. | 56. | .0 | 11.3 | .0 | 10.2 | 0. | 37. |

| | TOTAL PROPORTION | TOTAL STEMS/AC. | TOTAL B.A./AC. | ----D.B.H.----- MEAN | VARIANCE | AVERAGE HEIGHT | STAND AGE | SITE INDEX | T.C.F. VOLUME | M.C.F. VOLUME | B.F.S. VOLUME |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ACTUAL | 1.000000 | 1153 | 138.5 | 4.46 | 2.1327 | 37.8 | 44. | 82. | 2480. | 1339. | 605. |
| PREDICTED | .998817 | 1151 | 139.2 | | | 38.7 | | | 2517. | 1301. | 850. |

FISHERS KAPPA TEST
FAIL TO REJECT NULL HYPOTHESIS OF A NORMAL DIAMETER DISTRIBUTION AT P = 0.01.
ASSUME THAT DIAMETERS ARE DISTRIBUTED NORMALLY WITHIN THE STAND.
PROCESSING OF STAND WILL PROCEED.

STAND TITLE: TEST RUN WITH TEST.DAT INPUT DATA

TEN YEAR PROJECTION NUMBER 1

| | AGE | S.I. | MEAN D.B.H. | VARIANCE | MEAN HEIGHT | B.A./AC. | STEMS/AC. |
|---|---|---|---|---|---|---|---|
| STAND CHARACTERISTICS: | 54. | 82. | 5.1 | 2.2276 | 44.6 | 161.58 | 1030. |

| DIAMETER CLASS | NORMAL PROBABILITY | STEMS PER ACRE | CLASS HEIGHT | T.C.F. VOLUME | M.C.F. VOLUME | B.F.S. VOLUME |
|---|---|---|---|---|---|---|
| 0 | .0024733 | 3. | 7. | .0 | .0 | 0. |
| 1 | .0148687 | 15. | 16. | 1.2 | .0 | 0. |
| 2 | .0579486 | 60. | 25. | 21.1 | .0 | 0. |
| 3 | .1464140 | 151. | 34. | 139.1 | .0 | 0. |
| 4 | .2399706 | 247. | 41. | 459.2 | .0 | 0. |
| 5 | .2552291 | 263. | 48. | 845.0 | 597.0 | 0. |
| 6 | .1761599 | 181. | 53. | 907.8 | 733.4 | 0. |
| 7 | .0788849 | 81. | 58. | 590.3 | 510.3 | 1368. |
| 8 | .0229052 | 24. | 63. | 241.1 | 216.6 | 745. |
| 9 | .0043123 | 4. | 65. | 52.0 | 47.8 | 185. |
| 10 | .0005279 | 1. | 65. | 15.9 | 14.8 | 61. |
| | --------- | ----- | | ----- | ------ | ----- |
| STAND TOTAL | .9996942 | 1030. | | 3273. | 2120. | 2359. |

SUB-MERCHANTABLE T.C.F. = 620.6

SUB-SAWLOG T.C.F. = 2373.4

SUB-SAWLOG M.C.F. = 1330.4

Mowrer, H. Todd. 1986. ASPNORM: A normal diameter distribution growth and yield model for aspen in the central Rocky Mountains. USDA Forest Service Research Paper RM–264, 12 p. Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colo.

Development of a normal diameter distribution growth and yield model for pure, even-aged, unthinned clones of aspen in the central Rocky Mountains is described, including testing for normality of diameter distributions, development of regression estimators for stand parameters, model validation, and model application.

**Keywords:** Mensuration, growth and yield, *Populus tremuloides*, computer modeling

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U.S. Department of Agriculture
Forest Service

# Rocky Mountain Forest and Range Experiment Station

Rocky Mountains

Southwest

Great Plains

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

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